Self-Erecting Inverted Pendulum: Swing Up and Stabilization

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Abstract

Inverted pendulum control is a well-known and challenging problem, which is generally associated to attitude control of a rocket during take off. When upright the pendulum is at an unstable equilibrium point and constant attitude adjustments are required to maintain proper orientation. This problem is like trying to balance a broomstick at the tip of one finger. This nonlinear one-input, two-output system consists of a slender pendulum rod attached passively to a cart on a rack and pinion system driven by an electric motor on a horizontal axis. The objective is to swing the pendulum upright and maintain this angular position while satisfying a specified linear cart position by adjusting the terminal voltage to the motor. This objective is achieved by real-time control implemented with a Visual Basic computer program.

Sommaire

Le contrôle du pendule inversé est un problème très connu et stimulant, qui est d'habitude associé au contrôle d'attitude d'une fusée pendant le lancement. Quand le pendule est dans une position verticale, il est considéré dans une position d'équilibre instable et des ajustements constants de l'attitude sont nécessaires pour maintenir une propre orientation. Ce problème est similaire à celui de faire tenir en équilibre un manche à balai sur la pointe d'un doigt. Ce system non linéaire à entrées et deux sorties est constitué d'un pendule mince, attaché passivement à un chariot sur une crémaillère qui fonctionne par moteur électrique sur un axe horizontal. L'objectif est de balancer le pendule jusqu'à ce qu'il soit en position verticale et de le maintenir dans cette position tout en spécifiant la position linéaire désirée du chariot. Cet objectif est réalisé par un contrôle en temps réel implémenté via Visual Basic.

1.0 Introduction

The self-erecting inverted pendulum system can be used as an educational tool for control engineering students. The physical system is simple allowing an easy link to be made between the mathematical model and the real world. However, this system can also pose a challenging nonlinear control problem for students at the graduate level.

A simple yet functional graphical user interface (GUI) gives users the ability to explore differences created by changing settings such as sampling frequency and filter cutoff frequencies in real-time by the click of a button. This allows the user to transform mathematical theories into something tangible, ultimately leading to a better understanding of the interactions between theory and the physical world. The state feedback gains and the integral gain may be changed on the main form by simply typing in the desired numerical values. If changes are made while the system is running in real-time, the effects on the system performance are seen immediately.

Since the controller design is based on a linearized model, program modifications must be made to test nonlinear controller designs. However, with some programming skills and reference to a Visual Basic book [1], the documented and organized code may be altered to accommodate these changes.

The self-erecting inverted pendulum can be manufactured for a small cost making this an ideal laboratory experiment for undergraduate students studying physics, system modeling or control engineering.

2.0 Problem Formulation

The control objectives for the self-erecting inverted pendulum include swinging the pendulum rod into the upright position and then maintaining the rod in this position while holding a specified cart position.

The control design for the swing up of the rod will be separate from the control design for the stabilization of the cart and pendulum. The only objective for the open loop swing up controller will be to upright the pendulum. Once the angular position of the rod reaches a specified capture range the closed loop stabilization controller will take over. However, challenges lie in swinging up the rod. If the rod reaches the upright position with a high angular velocity, the controller designed for stabilization will not be able to compensate.

3.0 Control Scheme

Swinging the pendulum rod upright using minimal energy is achieved when the oscillating control input frequency is the natural frequency w_n of the rod. The natural frequency of the rod is obtained through calculation and measurement. The open loop control function is given by

$$V = A \sin (w_n t) \qquad (1)$$

This function has two software adjustable values, the natural frequency w_n and the gain A, which allows different rod lengths to be tested with the system.

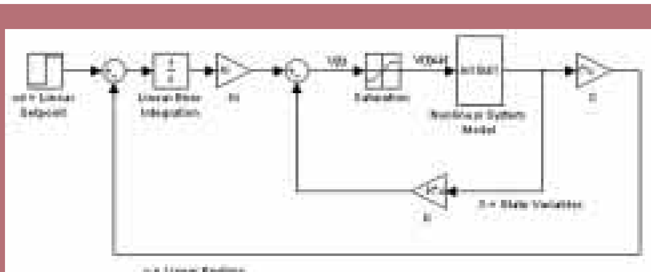

Figure 1: Closed Loop System

The stabilization control design is based on linear quadratic regulator (LQR) design with a tracking controller [2]. The LQR design will return the state feedback gains needed to ensure stability of the system. However, to bring the steady state error of the linear position to zero, a tracking controller is added by integrating the error of the cart position. The gain adjustment of the integration allows performance changes to be made. The control law implemented is given as follows

$$V(t) = -KX(t) + Ki\int_0^t (x_p - x)dt \qquad (2)$$

The closed loop system is shown in Figure 1. From this block diagram it can be seen that the control design uses state feedback. The gain values for the state variables are denoted by K, a 1x4 vector, and are the desired values to achieve stabilization and good performance. The gain block for the integration of the linear position error is denoted by Ki and is also the desired value to achieve near zero steady state linear position error and good performance. The saturation block is necessary to represent the experimental system as accurately as possible. Since $V(t)$, the calculated motor voltage, may reach higher than acceptable values for the motor, a saturation function is embedded in the software clipping the control effort, or motor voltage signal.

4.0 Experimental Setup: Physical System

The mechanical components, all made from lightweight aluminum, consist of a cart and gearing system, pendulum rod and track. The cart guide-rail and rotating pendulum rod shaft are made from stainless steel, ensuring strength and integrity. The position sensing electronics include a 10-turn potentiometer for measurement of the linear cart position and 2-channel optical encoder for measurement of the pendulum rod angle. The motor used is a high-speed DC permanent magnet mini motor. Combined with a large drive shaft gear the speed is significantly reduced while increasing the torque.

Acquisition of the sensor signals requires only one analog input for the cart position potentiometer and an up/down counter for the pendulum rod optical encoder. The control signal from the computer requires only one analog output fed through a buffer for current amplification to the motor.

The system has been designed so multiple pendulum rods of different lengths are easily interchanged. The difference in rod length and weight requires new gains to be calculated for the controller giving insight into system limitations and optimal performance.

Figure 2: GUI Main Form

5.0 Experimental Setup: Software Interface

The program created to control the self-erecting inverted pendulum was designed using Visual Basic for the GUI and rapid modification and design capabilities. The user can swing the pendulum upright with a command click or manually bring the pendulum upright by hand. When the pendulum rod is within a user specified inner capture range the closed loop controller will maintain stability of the pendulum rod.

Safety limits have been implemented that zero the control signal to the motor for four unique conditions. These conditions are as follows

1. The 'stop' command button is clicked.
2. The program is exited.
3. The pendulum rod violates the maximum angular position greater than the user specified outer capture range.
4. The cart reaches a plus or minus linear position greater than the user specified cart shutdown limit.

Figure 2 shows the GUI main form. This design allows for input of control parameters and displays measured and calculated feedback information to the user. The 'Active X' universal circular gauge and inverted pendulum model created for a simple inverted pendulum have been modified for this interface. The original 'Active X' controls are described in [3].

The user may also alter the system settings including base addressing, sampling time and inner and outer capture range for the pendulum angle. Other settings include safeties such as a motor shut down upon a critical linear position. This can potentially avoid damage to the linear position sensor and the mechanical system itself. Other adjustable settings include the swing up frequency and gain.

Digital filter settings allow the user to control the cutoff frequency of the linear position, angular position and the motor output. Each filter uses a first order discrete equation programmed into the software. The filtering of the input measurements from the linear position sensor and the angular position sensor are imperative for accurate readings. Due to the nature of the controller, any measured noise can affect the stability of the pendulum rod, and the position of the cart.

6.0 Simulation Results

The Matlab simulation will only test the pendulum when in the upright position. However, it will consider initial angular and linear positions as well as disturbances, and allow linear set point changes. When simulating the controller performance it is important to represent the system with the nonlinear equations. This is necessary to obtain results as close as possible to the real system. The block diagram of the nonlinear system representation, shown in Figure 3, was designed using Simulink because of the rapid model-based design capabilities and quick modifications for simulations [4].

The simulation is performed using calculated values for *K* and *Ki*. The simulation parameters are given below:

- Initial Angular Position: 0.398 radians
- Linear Set Point (initially 0m): 0.2m at time 7.5s
- Disturbance Introduction: 0.2 radians at time 5s
- Disturbance Cancellation: -0.2 radians at time 5.2s
- Simulation Time: 20s

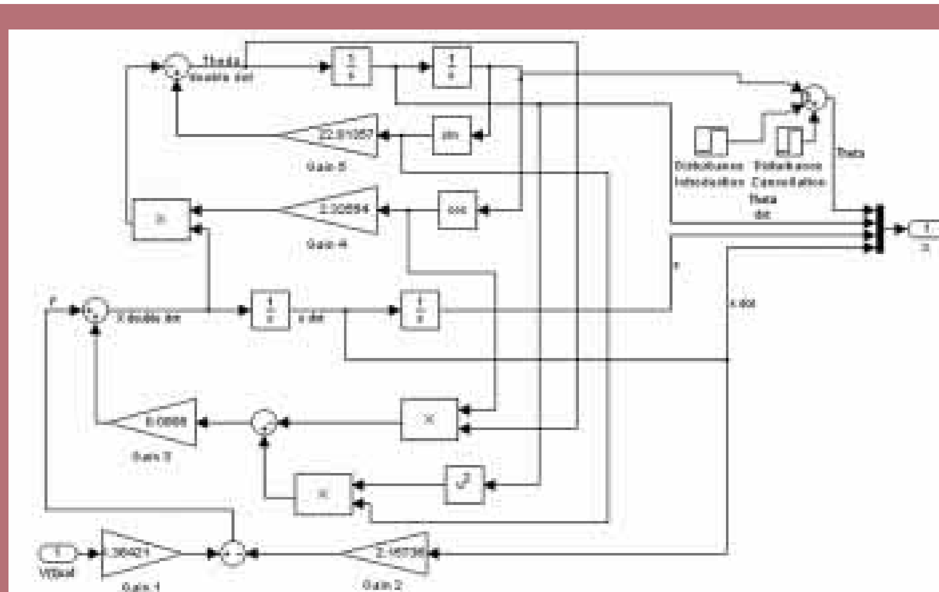

Figure 3: Nonlinear System Representation

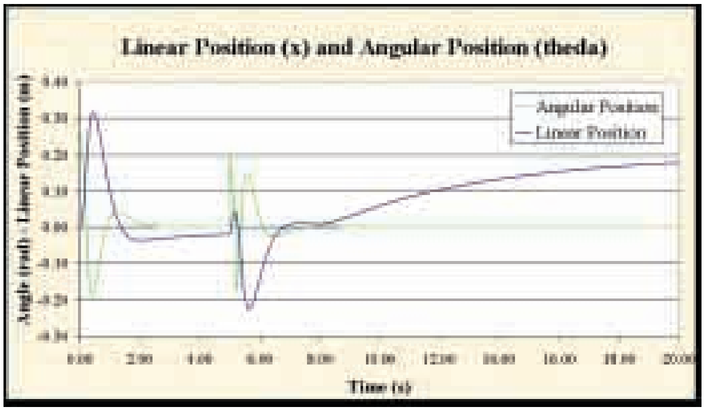

Figure 4: Simulated Response

The results obtained from the simulation are seen in Figure 4 and Figure 5. It is apparent that the calculated motor voltage has been limited to a ±5 Volt limit to more accurately represent the physical system. For the physical system the voltage saturation is necessary to avoid drawing too much current during swing up, eliminate possibilities of exceeding maximum terminal voltage and to achieve a better response from the motor.

From the results of the simulation it can be seen that at time 5 seconds, a disturbance to the angular position was introduced and cancelled 0.2 seconds later. To compensate the controller responded accordingly, changing the linear position and subsequently the angular position and maintained stability.

It can also be seen that at time 7.5 seconds, the desired linear position set point changed from 0m to 0.2m and the corresponding linear position gradually began to converge to the set point. However, even after a simulation time of 20 seconds the linear position did not quite meet the desired set point. This implies that a larger integral gain *Ki* is required to reduce this convergence time.

7.0 Experimental Results

To ensure an accurate comparison between the simulated results and the experimental results, the same state feedback gain *K* and integral gain *Ki* were used for experimental testing. The results can be seen in Figure 6 and Figure 7.

The system parameters are slightly different from the simulation in that the initial angular position is the stable equilibrium point at 180°. Additionally, the swing up control is used to erect the pendulum in the upright position.

The system gain *A* is shown to be greater than the ±5 Volt saturation cutoff limit of the simulation. The software has been programmed with two terminal voltage saturation points. The first is set to ±7.5 V during swing up, and the second is set to ±5 V once the stabilization controller initiates. This is necessary to achieve a cart driving force great enough to erect the pendulum rod. At the ±5 V limit, the motor is not able to produce enough energy to swing the rod upright. However, using the ±7.5 V limit during stabilization can cause the system to destabilize.

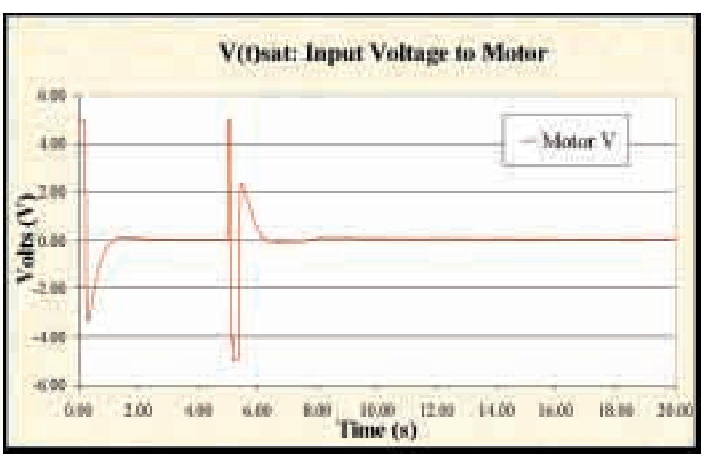

Figure 5: Simulated Control Effort

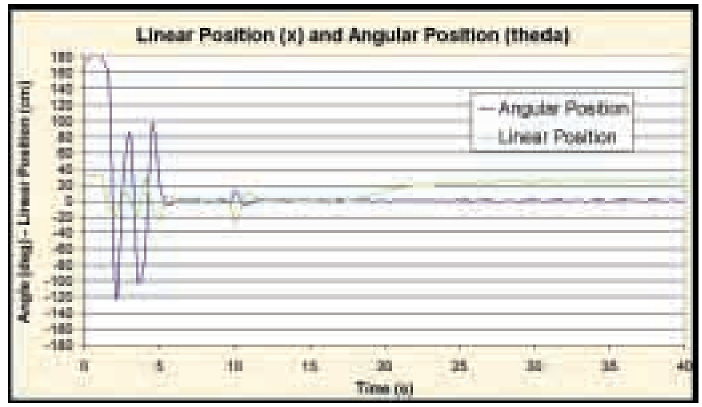

Figure 6: Experimental Response

Once the swing up controller erected the pendulum, the stabilization controller initiated at roughly 5.5 seconds. After stabilization was achieved a disturbance was introduced to the pendulum rod at about 10 seconds. This disturbance was introduced by 'tapping' the pendulum rod in one direction with a force from the hand. Also at roughly 17 seconds the linear set point was changed from 0 cm to 25 cm.

When viewing the experimental results, there are a number of observations to be made. It is important to note the angular position response of the physical system during swing up. It appears that the angle is crossing the 0° threshold on each pass of the cart. This would imply a full revolution of the pendulum rod. However, this is not the case. When viewing the GUI main form, one can see the angular position measurement has been set up with 0° at the top, and 180° and -180° both meeting at the bottom of the circle. Due to this configuration, as the pendulum swings through the bottom of the measurement circle the system interprets this as a full revolution due to the digital filter on the angular position.

The calculated motor voltage seen in Figure 7 shows a noisy response. Even with the digital filter added to the output of this calculation, there is a substantial amount of calculation noise present. However, it is important to note the peak values at different times during this test. When a disturbance was introduced at 10 seconds, the motor voltage spiked up to roughly 5 V. The other various spikes in the response of

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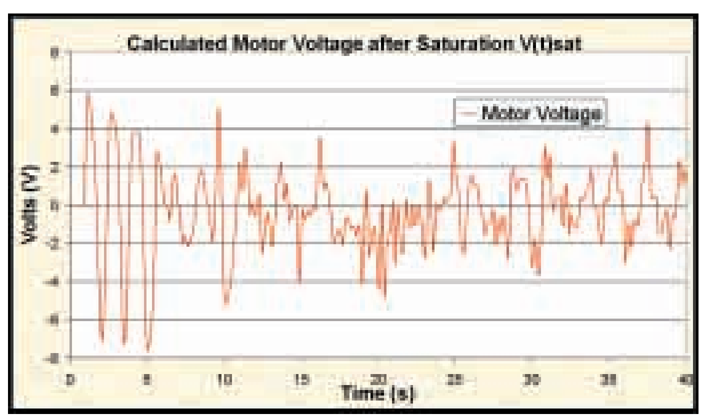

Figure 7: Experimental Control Effort

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the motor voltage after this disturbance do not peak much over 4 V at times. The response of the motor is the important factor for this plot seen. The motor does not begin to generate much driving force until it nears the ±4 V range.

8.0 Conclusion

The self-erecting inverted pendulum has been manufactured for a small cost and experiments have shown promising results. This system can be used as an educational tool for helping understand model dynamics and controller response.

Suggested improvements for this system would include a motor with an improved response. This would increase the robustness of the system and enhance disturbance recovery and swing up.


Image of the pendulum

After many experimental tests the repeatability of the swing up controller is less than ideal. Since this controller is open loop, any disturbances such as slight bends in the electrical harness attached to the cart create friction causing the system to respond differently each time. Improvements could be implemented by designing a closed loop controller for the swing up.

9.0 Acknowledgments

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10.0 References

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About the author

Stephen McGilvray received his B.Eng degree in Electrical Engineering from Lakehead University in 2002. He was awarded the IEEE Life Member Award in 2003 for his student paper on control of a self-erecting inverted pendulum. He is currently working towards his M.Sc.Eng. in Control Engineering at Lakehead University in Thunder Bay. His main areas of research include nonlinear control of a VTOL four-rotor helicopter and force control of robot manipulators.