Passivity as a Framework for Design and Analysis of Networked Systems: From Power Systems to Formation Flight

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Abstract

Large interconnected dynamical systems characterize many engineering, biological, and societal systems and are expected to be omnipresent in future technologies. Distributed control of electrical power systems, human neural networks, and emerging collective behaviors are example of complex systems whose understanding, although intricate, is fundamental to prediction and control purposes. Analyzing condition of stability of equilibrium for a given system is often a prerequisite in the derivation of mechanisms that allow achieving a desired behavior. The passivity approach, which is reminiscent of circuit theory, is reviewed as a mean to analyze stability of interconnected systems and to design distributed controllers that use local information. It is shown, by means of examples of a power system and a formation of autonomous vehicles, how stability can be warranted from an energy-balance consideration known as passivity.

Sommaire

De nombreux phénomènes, qu'ils soient d'ordre sociétal, biologique ou technologique, résultent de la mise en réseau de systèmes dynamiques. Les réseaux électriques, les réseaux de neurones humains ou l'émergence de comportements collectifs appartiennent à une classe de systèmes que l'on peut qualifier de complexe. Bien que difficile, leur compréhension est néanmoins requise si l'on souhaite prédire et maîtriser leur comportement. Ce faisant, la stabilité du ou des points d'équilibres de tels systèmes est une des notions importantes à considérer. Cet article se propose de revoir le potentiel que présente l'approche de passivité dans l'analyse de la stabilité et la synthèse de commande décentralisée de certains réseaux. Provenant initialement de la théorie des circuits électriques, le formalisme énergétique propre à la passivité permet d'appréhender avec succès l'analyse de certains réseaux tels que les réseaux électriques et le groupement de véhicules autonomes.

1.0 Introduction

Interconnected power systems, control of autonomous vehicles for defence applications [1], and control of communication networks [2] are among complex adaptive networked systems [3] that are either in use today or are emerging and expected to be pervasive technologies in a not so distant future. Such systems are characterized by multiple, possibly simple, and adaptive agents, which are distributedly controlled by feedback of local information. Components of these networked systems may be geographically dispersed and evolving in a competing or cooperating environment. In an environment prompt to rapid changes, distributed control offers the advantages of complying with limited data-rate communication and bounded computation capabilities, and of being more reliable to component failure than centralized control and decision making processes.

However, obtaining a clear understanding of the behavior of networked systems often remains a difficult task. In particular, achieving performance requirements must be accompanied with a guarantee of stability around a desired behavior. Among techniques that allow dynamical system analysis, passivity is an interesting approach to stability analysis of multi-agent dynamical systems for its invariance property through the feedback interconnection of any number of systems. Passivity provides the engineer with a powerful tool for nonlinear systems stability analysis and control synthesis. Passivity-based stabilization of dynamical systems has been investigated quite extensively over the last thirty years [4]-[6]. Induction motors [7], robots [8], smart actuators [9], and haptic environments [10] are among the applications that have benefited from passivity.

We present passivity as a framework for the design and the analysis of networked systems, giving application examples of power systems and formation flight controllers. The basics of passivity are explained, then a general framework for analyzing interconnected systems is described.

2.0 Limitation factors

One of the first results on passivity dates back to the 1950s, where the connection between passivity and stability of linear networks was established by the work of Youla *et al.*, [11], in the context of circuit theory. Passivity can be introduced by considering the RLC circuit of Figure 1.

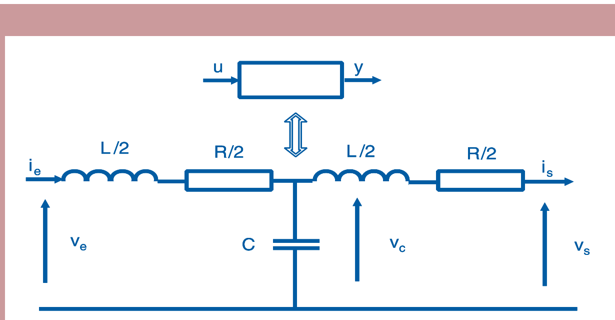

Figure 1: RLC Circuit

Kirchhoff's laws and simple algebraic manipulations lead to the following energy-balance equation:

$$(1)\quad \underbrace{\int_{t_0}^{t}(v_e(\tau)i_e(\tau)-v_s(\tau)i_s(\tau))\,d\tau}_{\text{Supplied and delivered energy}}$$

$$=\underbrace{S(t)-S(t_0)}_{\text{Stored energy}}+\underbrace{\frac{R}{2}\int_{t_0}^{t}(i_e^2(\tau)+i_s^2(\tau))\,d\tau}_{\text{Dissipated energy}}$$

The input-output pair $(u,y)=((v_e\,,-i_s),(i_e,v_s))$ is said to be passive with storage function $S(t)=\frac{L}{4}\left(i_e^2+i_s^2\right)+\frac{C}{2}v_c^2$ and with dissipation in current. More generally, for lumped multi-input multi-output nonlinear systems Σ, passivity expresses an energy-like balance for input-output pair (u,y) characterized by:

$$(2)\quad \underbrace{\int_{t_0}^{t}u^T(\tau)\,y(\tau)d\tau}_{\text{Supplied energy}}=\underbrace{S(t)-S(t_0)}_{\text{Stored energy}}+\underbrace{\delta\int_{t_0}^{t}\|y(\tau)\|^2d\tau+\varepsilon\int_{t_0}^{t}\|u(\tau)\|^2d\tau}_{\text{Dissipated energy}}$$

If δ and ε are zero, the system is lossless. If $\delta>0$ (respectively, $\varepsilon>0$), the system is strictly output passive (respectively, strictly input passive); that is, dissipation occurs at the output or the input, or both. In other words, a passive system is a system that cannot store more energy than supplied.

A memoryless nonlinearity restricted to the first and third quadrants, as illustrated in Figure 2(a), is passive if the *u*-axis is included in the function definition space and strictly passive otherwise. This can be shown by using (2), given that the product of u and y is always positive and that by definition the stored energy for this element is zero, so that energy is dissipated at all time, unless u or y equals zero. Henceforth, V-I characteristic of a diode and saturation characteristic of a magnetic circuit (without hysteresis) are examples of memoryless passive component models.

For a dynamic model, the phase angle of passive (respectively strictly passive) linear systems is within $[-\pi/2 \text{ rad}, \pi/2 \text{ rad}]$ (respectively, $(-\pi/2 \text{ rad}, \pi/2 \text{ rad})$). A complete set of passivity (positive realness) conditions is presented in [12]. Hence, the negative feedback of two strictly passive systems, Σ_1 and Σ_2, has a phase angle less than 180° and is characterized by an infinite gain margin as shown in Fig 2(c).

Relationships between passivity and stability are fundamental results that are well known in the fields of nonlinear systems [4]. Energy-balance inequality (2) with dissipation suggests that one expects stability or at least stabilizability of Σ. From the use of inner product $u^T y$ in (2), passivity is naturally geared to the space (L_2) of finite energy signals. More precisely, from (2) and condition $\delta > 0$, it can be shown that strictly output passive systems are bounded-input bounded-output (BIBO) stable in the L_2 space, as illustrated in Figure 2(b). This means that such systems have finite input-output gains. Furthermore, a connection with the internal stability of systems, that is, stability of the states around an equilibrium, can be established provided some form of observability or detectability is met [5].

Circuit theory can be helpful for the understanding of passivity invariance results [13]. For instance, series and parallel connections of passive electrical components, such as resistor, capacitor, and inductor, remain passive. Furthermore, Tellegen's Theorem [13] implies that a network made up of passive N-ports will itself be passive. Equivalent results in mechanics, for instance, can also be found by considering mass, spring, and dashpot. Extension of these facts to nonlinear systems is possible by means of the passivity theorem, which states that the feedback connection of two passive systems is passive [4]-[5]. Several other versions exist that relate strict passivity to input-output stability. Roughly speaking, these theorems result in the invariance of stability or, at least, stabilizability of passive systems that are in a feedback interconnection. This property is particularly well suited to analyze the behavior of networked systems that can be represented as a feedback interconnection of passive or to-be-passivated subsystems [14].

3.0 Passivity and Networked Systems

Networks of dynamical systems are generally represented as sets of ordinary or partial differential equations and a matrix H of operators K_i that models the interconnection structure of the network. K_i is typically used to model the dynamics between two adjacent subsystems that we indistinctively call nodes or agents. For instance, in the context of electrical network, K_i can be a function of impedances between a node i and its adjacent nodes j. The matrix H is often related to the generalized Laplacian of the graph, [15], that characterizes the relationship between neighboring nodes of a network. Adjacency of each node and directedness of edges ($i \rightsquigarrow j$ for directed edge), as illustrated in Figure 3(a), are information embedded in H.

From the structure of Figure 3(b) and applying the passivity theorem, stability of the networked system is obtained if the feedforward-path subsystem is passive and the feedback-path subsystem is strictly passive. Depending on the control system's degrees-of-freedom, forward-path and feedback-path subsystems can be rendered passive, if not already, provided some structural properties of each subsystem is satisfied [5].

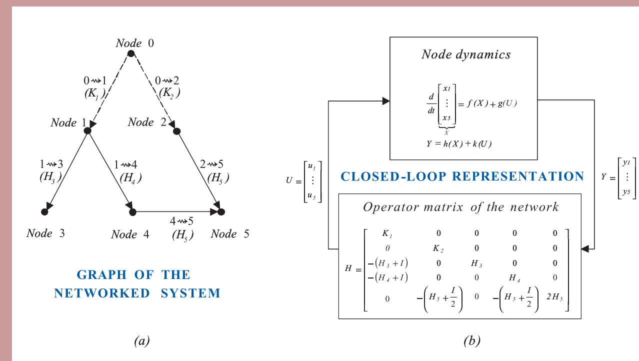

Figure 3: Graph and closed-loop representation of a network system

Passivity techniques have been recently used to solve stabilization problem of multi-agent systems. For instance, the interpretation of an optimization-based network flow control as a closed-loop system and the use of passivity can be used to prove the stability of a class of network flow regulation, which is typical of internet congestion control [16]. This approach has also been used for the decentralized power control of code division multiple access systems. A gradient-based law, which is obtained from a game-theoretical formulation, is shown to converge to a Nash equilibrium by means of passivity argument applied to a closed-loop model that results from the feedback interpretation of an optimization problem [17].

4.0 Passivity for the Stability Analysis of Electrical Power Systems

Passivity can be used to analyze power systems stability when the networked system is faced with: (i) voltage disturbance $\tilde{\mathbf{v}}_o$ located at an observation point or a connection point o to another subnetwork; (ii) disturbance $\tilde{\mathbf{v}}_{cj}^{\delta}$ at a point where the electrical component C_j is connected to the power grid (Figure 4). In order to work within an input-output point of view, the power system is decomposed by using the component-oriented modeling technique [18]. By component is meant an electrical load, a generator or a compensator. Figure 3(b) suggests an obvious closed-loop interpretation for electrical power network, where the power grid is represented by an admittance matrix H. Each component connected to the network is modeled as a voltage or current source in feedback with H as shown in Figure 4. Each component or aggregate of components is supposed to be controllable through channels $\mathbf{u}_{vi}$ and $\mathbf{u}_{ci}$. When no perturbation occurs, the system lies in its equilibrium point ${}^0x$; when perturbations occur, the system is described with error signals $\tilde{x}=x-{}^0x$, where x is any current i and voltage v of the network.

Assume the power grid can be approximated as a linear time-invariant N-port. To ensure strict passivity of the forward-path subsystem of Figure 4, the complex admittance $H(j\omega)$ has to verify the following strict-positive realness constraint $H(s-\varepsilon) + H^*(s-\varepsilon) \geq 0$ for $\text{Re}(s) \geq 0$ and some $\varepsilon > 0$.

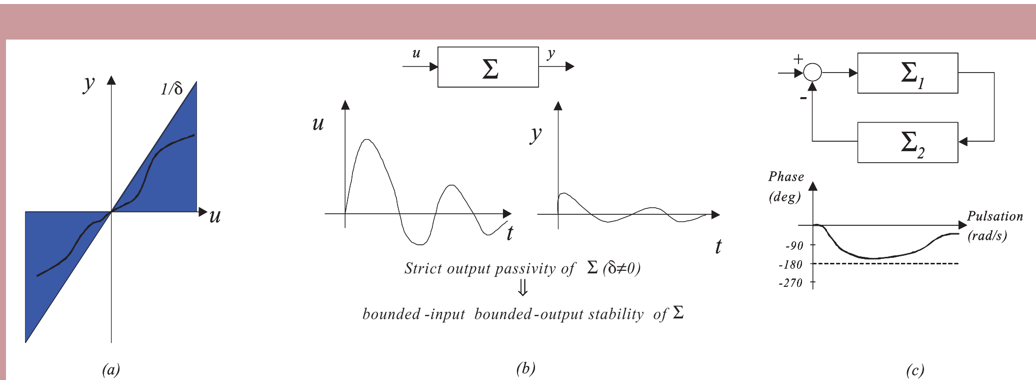

Figure 2: Input-output sector and energy properties

It was proven in [19] that a radial power network whose lines are represented with the T-equivalent model shown for one phase in Figure 1, is strictly input passive with only current source components. A generalization to both types of components in feedback necessitates considering small parasitic shunt resistances in parallel with C_j.

A direct application of passivity theorem indicates that finite-energy stability is obtained if each component or aggregate of components is passive or has

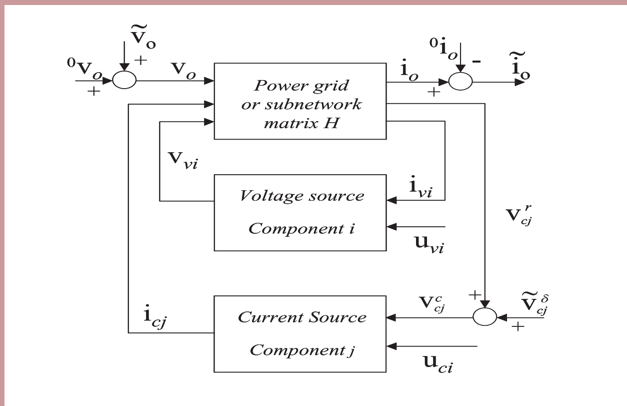

Figure 4: Component-oriented modelling of an electrical power system

been passivated by means of control actions $\mathbf{u}_{vi}$ and $\mathbf{u}_{ci}$. If the disturbance vanishes or the fault is cleared, asymptotic convergence of the network state can also be shown [19]. Article [19] and references therein draw a list of passivated electrical components such as turbo-alternators, induction motors, and a class of FACTS, namely, STATCOM, which could be used to passivate a load aggregate rather than rendering each and every single load passive. Conditions can also be given so that specific classes of aggregate loads, large motor, thermostatic heater, and on-load tap changer are passive or quasipassive where, in the latter case, sector condition related to passivity is lost in a region containing the equilibrium.

Component C_j is not limited to being a single electrical apparatus. Indeed, the network of Figure 4 could be connected at point *o* to another network by means of admittance matrix *H*, which would have to satisfy condition (3). Furthermore, if some components or aggregate of components are not, or cannot, be made passive, weaker stability such as BIBO in magnitude ($L\infty$ space) may be obtained provided some form of quasipassivity is observed [19].

There exist classes of mechanical systems, such as robots, that are naturally found passive from force or torque input to speed or angular rate output. Equilibrium is characterized by zero speed, which means that the amount of energy necessary to steer the perturbed system to an equilibrium is finite. It is not necessarily so with electric systems whose equilibrium is not characterized by zero current or zero voltage. Passivity is therefore applied to error dynamics whose equilibrium is $\tilde{x} = 0$. Other approach and passivation schemes, which circumvent the finite dissipation obstacle, are discussed in [20], [21].

5.0 *Passivity for Formation Flight Control Design*

Designing decentralized controllers for a formation of autonomous vehicles can be tackled by means of passivity arguments. Each vehicle control only feeds back information from its neighbors such as relative distance and speed. Vehicles are considered neighbors to agent *i* as long as they are located in a region defined by their sensor range limits of *i*.

There exist several definitions for analyzing the stability of a formation of autonomous vehicles. Mesh stability is defined as the combination of the Lyapunov stability of interconnected systems with the input-output stability of inner subsystems [22]. String stability is the one-dimensional equivalent of mesh stability and is of interest mainly in automated highway systems [23].

In the context of leader-follower maneuvers, it is interesting to analyze the behavior of the formation when the leader tracks a smooth curve and to verify that stability is maintained in case of disturbance applied to the nodes. Achieving stable formation morphing can also be addressed by means of passivity. Simply stated, the concept of formation morphing is defined here as performing set-point regulation of the changing relative distances between neighboring vehicles with a time-invariant graph topology of the formation. Achieving stable morphing can be useful, for instance, in inspection tasks during which the vehicle formation has to expand and to contract its geometry to comply with the geometrical constraints imposed by the environment, such as when transiting from wide open areas to constrained spaces.

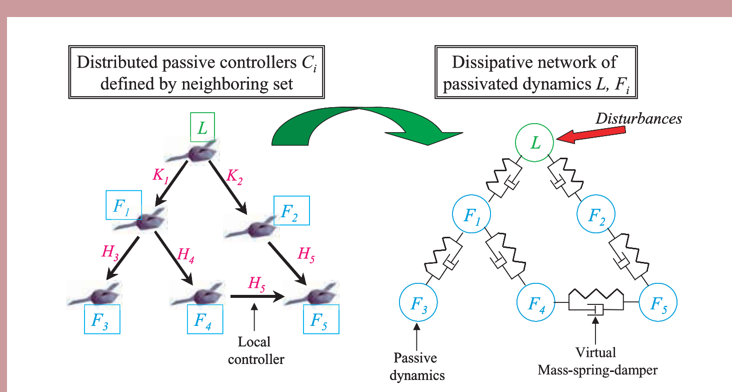

Figure 5: Physical analogy of formation flight control through passivity

As suggested by Figure 5, the formation dynamics can be decomposed into two classes of dynamics: (i) dynamics of nodes *L* and F_i; (ii) dynamics of controllers that virtually link two neighbors. A physical interpretation of such networked systems is given by the representation of interconnected vehicle dynamics as virtual springs and dampers shown in Figure 5. The spring-damper interpretation of the networked system allows the designer to adopt the closed-loop system viewpoint of Figure 3(b), where the matrix *H* embeds the interconnection structure, and the virtual mechanical components, which are represented by operators H_1, H_2, K_3, K_4, and K_5. The control law applied to each vehicle is composed of two loops. One loop is dedicated to the passivation of the local node. The other loop feeds back relative distance and speed between neighbors and is represented by one of the aforementioned operators, which are designed to render *H* strictly passive. These control schemes aim at achieving:

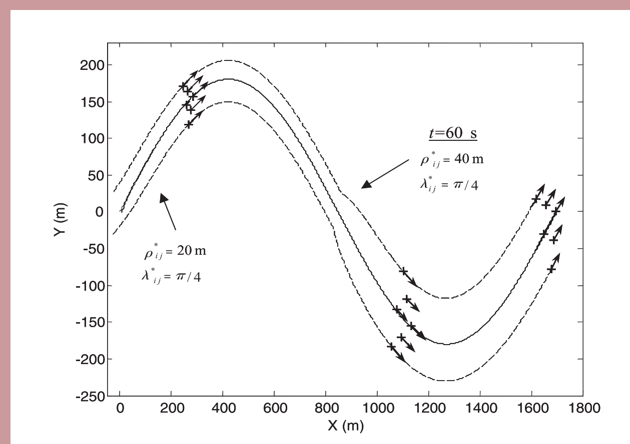

Figure 6: Time evolution of the formation with large relative position commands $\rho^*_{ij}(0) = 20$ m & $\rho^*_{ij}(60) = 40$ m

- stable trajectory tracking with respect to disturbances (leader and followers);
- robustness with respect to parametic uncertainties in the dynamics;
- stable piecewise-constant morphing of the formation.

As an example, a formation of six vehicles with the interconnection graph of Figure 5 is required to track a sinusoidal trajectory and to stabilize the inner relative positions among the vehicles at the range of 40 m at time *t*=60 s from an initial relative position of 20 m. For the sake of clarity, only the trajectory of the leader and of nodes 3 and 5 are shown in Figure 6.

The arrow represents the speed vector of each vehicle. It is shown that the decentralized two-loop passivity-based control law stabilizes the formation. More precisely, stable set-point regulation of inner relative distance and of line-of-sight angle between neighboring vehicles is achieved while the leader is asked to follow a smooth curve.

6.0 Prospective application to automated highway

A particular application on the 1D version of the leader-to-follower stability problem, which can be related to string stability [23], is the automated highway; see for instance the California PATH project in [24]. In some situations, car drivers perform decentralized control of their vehicle based on perception of their neighbor's behavior. A particular topic of interest would consist of analyzing, by means of passivity, the stability of a platoon of cars in response to an abrupt deceleration of the leader. Driver's reaction delay, too small inter-car separation along with high speed are among a set of conditions that are likely to lead to string instability of the platoon with potentially dramatic consequences such as car pile-ups downstream in the string. The use of appropriate slowdown warning systems and conditions to maintain safe relative distances despite abrupt contingencies could be derived by adopting a setting similar to the planar vehicle formation control.

7.0 Acknowledgement

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